Eleventh Semiannual Status Report

December 1966

on

BASIC STUDIES IN SPACE VEHICLE ATTITUDE CONTROL

in the

Department of Aeronautics and Astronautics
Stanford University

under

Research Grant NsG-133-61

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SUMMARY

The research on nonlinear and optimal control of planet-pointing space vehicles, begun by Messrs. Busch, Almuzara, and Hales, continues with new Ph.D. candidates under Professor Flügge-Lotz (Sec. A). Basic studies of optimal control, under Professor Franklin, and satellite trajectory studies, under Professor Breakwell, also continue (Secs. A and B).

In September, Professor Cannon began a one-year leave of absence from the University to act as Scientific Advisor to the Chief of Staff of the Air Force. Consequently, his research is suspended, and there is nothing to report for him on this Grant for the current period.

A. NONLINEAR STUDIES, OPTIMAL CONTROL
(Studies Supervised by Professor Flügge-Lotz)

1. Optimal and Suboptimal Control of a Satellite in Elliptic Orbit. Steady State Consideration. (Linearized Equations of Motion) (Ph.D. Research of W. Boykin)

Dr. R. Busch developed a good suboptimal acquisition control and has spent some thought on the steady state control [Ref. 3]. However, if keeping a certain fixed attitude over a long time (one or two years) at minimum fuel expense is desired, further studies were indicated. In view of the desired high accuracy, Mr. W. Boykin considered in detail the forces and their torques due to the control, the gravitational attraction of the earth and other celestial bodies, the interactions of the satellite with the earth's atmosphere and magnetic field, and the interactions of the satellite with emissions from the sun and with meteoroids.

A search was carried out for a feedback (and perhaps time-varying) control system which would solve the attitude station keeping problem with minimum control effort. A pointing accuracy of 10^{-3} radians (or less) with a rate of 10^{-6} radians per second (or less) was sought.

The search was conducted with the aid of Pontryagin's Maximum Principle, extensions of it, the theory of the stability of nonlinear systems, and high speed computational devices.

The maximum principle was applied to the system with various cost functionals which were functionals of the states as well as of the control effort.

A set of control laws was obtained. The control laws are simple enough for implementation in the near future.

2. The Complete Attitude Control Problem for an Earth Satellite in Elliptic Orbit. (Ph.D. Research of G. Wolske)

In the preceding Status Report it was indicated that the computation of the necessary control developed by Hales needed too much time and that the iteration procedure needed speeding up. A new Ph.D. candidate, Mr. G. Wolske, has started to investigate the available possibilities. Mr. Wolske has spent these months in getting acquainted

with all aspects of the problem: the dynamics, the control, the computational procedures. He studied additional literature to consider using second variations.

3. <u>The Validity of Linearization in Attitude Control.</u> (Ph.D. Research of F. Curtis)

The optimal control of two systems

I. $$\ddot{x} + \sin x = u(t)$$

II. $$\ddot{x} + x = u(t)$$

with $|u| \leq A \geq 1$

for the performance criterion $\int_0^T |u|\, dt \to \min$ shall be compared. The control of the linear system II is well known. Mr. Frank Curtis studies system I. He has been studying the optimal trajectories and switching sequences of the pitch equation $\ddot{x} + \sin x = u$ of a satellite with bounded control for minimizing the amount of fuel spent in zeroing position and velocity disturbances. The time allowed to zero an error has been restricted to $T \leq \pi$ seconds which corresponds to a requirement of less than half an orbit.

The concentration was on the backward time trajectories which cross the x_1 axis $(x = x_1)$ -- a singular line of the governing adjoint equations. Results include that at most two switchings occur before crossing this singular line having two possible switching sequences $+ A, 0; + A, 0, - A$.

In the extension of these trajectories into the upper half plane, the control may alternate between $- A$ and 0 with the periods of coasting being nearly centered about lines described by $\dot{x}_1 = x_2 = (2k + 1)\, \pi$. The time constraint $T \leq \pi$ limited backward time switching sequences to $[+ A, 0, - A, 0, - A]$ as the most general with a portion of this sequence being optimal for initial disturbances sufficiently near the origin. Whereas a complete time-varying feedback control law is yet to be found for the entire region under consideration, the possible sequences of control have been characterized.

4. Computation of Optimal Controls by a Method Based on Second
    Variations   (Ph.D. Research of T. E. Bullock)

In the Tenth Semiannual Status Report [Ref. 4], the development
of a control optimization program based on second variations was
described. The first stage of this development has now been completed
and a technical report giving details is in preparation. This report
is based on the doctoral research of T. E. Bullock, and will be issued
in the normal manner in the near future.

The final step in this particular research program consisted of
the successful computation of an optimal trajectory to a low dimension
terminal manifold. The example used for illustration of the method
was a modification of the problem using Van der Pol's equation originally
suggested by Merriam [Ref. 5]. In this problem the equations of motion
are given by

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 + (1 - x_1^2) x_2 + u$$

The initial conditions are taken to be $x_1(0) = 1$, $x_2(0) = 0$ and
the initial control guess is $u = 0$. The cost function is

$$J = 1/2 \int_0^5 (x_1^2 + x_2^2 + u^2)\, d\tau$$

and the final value constraint

$$\Psi = (1 - x_1 + x_2)\big|_5 = 0$$

is imposed.

The constraint, which is a scalar, requires that the trajectory
terminate on a <u>line</u> in the x_1, x_2 plane. This requires that one of
the adjoint variables in the variational problem is fixed and the
other unknown at the terminal time. Furthermore, the solution to the

matrix Riccati equation needed to construct the second variations becomes singular and unbounded at the final time. The development of a procedure with "second order" convergence in this situation is the main result of this research.

Trajectories of the computer iterations are shown in Fig. 1. As can be seen from the figure, convergence is obtained in six steps from an initial guess which is quite far off. A conjugate point in the auxiliary problem was found on the second (2) iteration which was removed in the third. This is indicated by the fact that these two trajectories have the same initial segments but separate in the final part, the separation taking place near the x_2 axis in this case. As is typical of this second-order method, a feedback control for neighboring trajectories is automatically generated by this program. The coefficients of this control are given in Fig. 2.
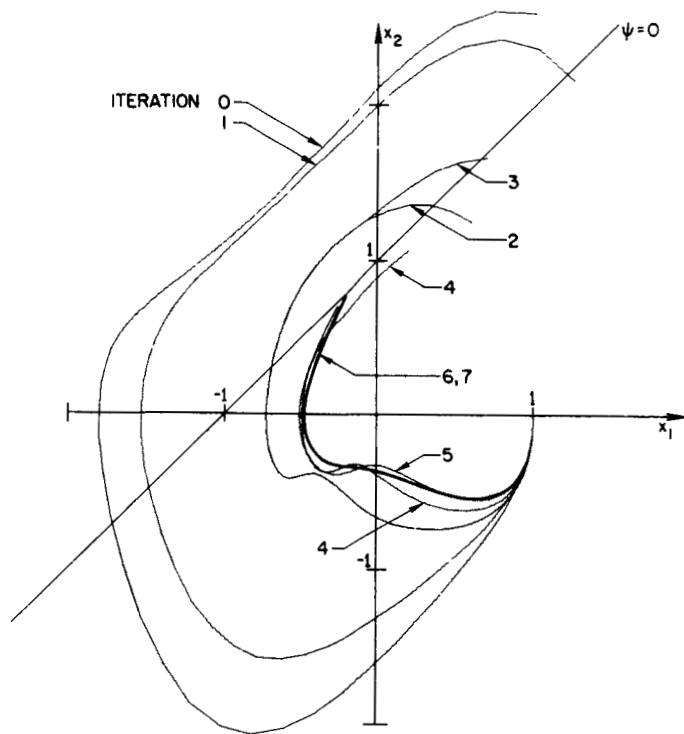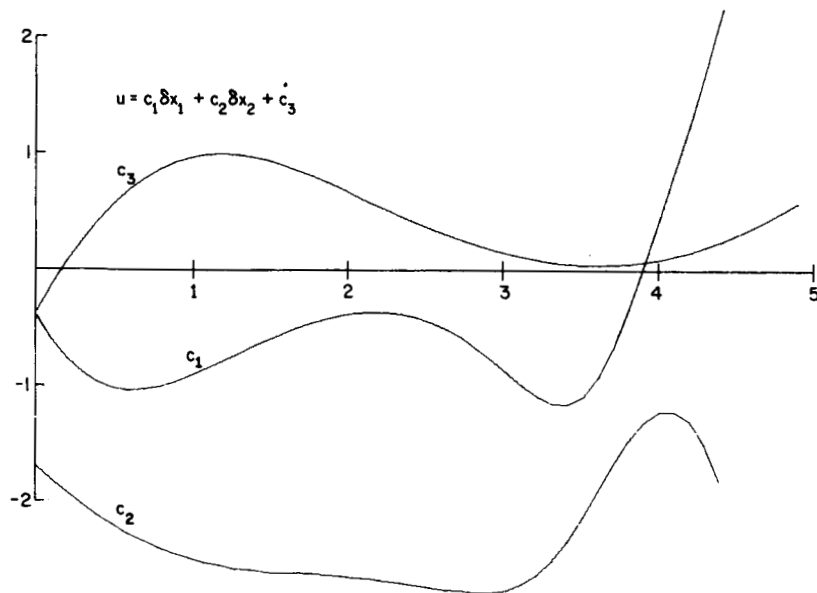
FIG. 1.   ITERATIONS IN OPTIMAL CONTROL COMPUTATIONS

FIG. 2.   FEEDBACK CONTROL COEFFICIENTS

B. SATELLITE ORBIT STUDIES
(Studies Supervised by Professor Breakwell)

1.   Earth-Satellite Orbits in Resonance with a Tesseral Perturbation
(Ph.D. Research of J. Vagners)

An essentially circular orbit, perturbed by tesseral as well as
zonal harmonics of the earth's gravitational field, will undergo
resonant perturbations if the mean motion is sufficiently close to an
exact multiple of the earth's rotation rate. A well-known analogy with
the classical nonlinear pendulum shows that in-track position can contain
a large long-period term accompanied by a small (i.e., $\sim 10^{-3}$) long-
period fluctuation in major-axis.

When a nonzero eccentricity is included, the usual long-period
fluctuations due to zonal harmonics must be modified to account for
the above major-axis fluctuation as well as for eccentricity terms in
the tesseral perturbing function (carried, for example, through e^2).
To obtain explicit formulas for the corrected long-period behavior,
the elliptic functions which describe the pendulum-like oscillation
in in-track position and major-axis are expanded in Fourier series,
except in the neighborhood of the "separatrix" which separates libration
from circulation, in which neighborhood the fluctuations essentially
come to a halt near the unstable equilibrium, and hyperbolic functions
of time are convenient.

2.   Motion Near Earth-Moon L_4 Libration Point   (Ph.D. Research of
H. Schechter)

A modification has been found of the Breakwell-Pringle procedure
[Ref. 6] for analyzing the main effects of nonlinear terms in the
equations for motion near L_4. The modification concerns the manner
in which short-period oscillatory terms are removed from the disturbing
function, leaving only near-resonant terms arising either from the
near-commensurability of the linearized free oscillations (internal
resonance) or from near-commensurability with the dominant solar
perturbation (external resonance). The modified procedure for the
internal problem insures that the short-period position and velocity

corrections are <u>polynomials</u> in the linearized expressions, as in the recent work [Ref. 7] by Deprit, Henrard and Rom. Indeed, subsequent removal of the remaining (long-period) oscillatory terms will agree up to second order with Deprit's double d'Alembert series for the two-dimensional problem without solar perturbation. The modified computations for periodic orbits near L_4 in the restricted problem are expected to agree rather well, at least for those orbits which remain within say 50,000 km of L_4, with the numerical investigations of Deprit. Such was not the case for the orbits computed in the original Breakwell-Pringle paper [Ref. 6].

When the solar perturbation is included, a <u>stable</u> equilibrium in the long-period variables has now been found for a two-dimensional description, in which the slower free mode is not present. Computations are in progress for the nonlinear coupling terms with the out-of-plane motion, also excited by the sun. If the stable equilibrium of the two-dimensional model carries over to the (correct) three-dimensional description, nearby points in the phase-space will correspond to initial conditions leading to motions which remain in some neighborhood of L_4 for an indefinite period of time, at least if the corresponding linearized motion is not excessively large.

C.   OTHER ACTIVITIES

Professor Cannon has taken a one-year leave of absence to act as Scientific Advisor to the Chief of Staff of the Air Force.   He will resume his responsibilities at the University in September 1967.

The four professors participating in this grant all attended the AIAA Specialist Conference in Guidance and Control and the Joint Automatic Control Conference in Seattle in August of this year.   One Stanford paper was presented at those meetings [Ref. 8].   Other research will be reported in the usual manner [Ref. 9,10].

Professors Flügge-Lotz and Franklin attended the Contractor's Symposium on Control Theory at MSFC, Huntsville, September 19-21.   The symposium was sponsored by the Astrionics Laboratory directed by Dr. Haeussermann.   Dr. J. H. George was the coordinator.   Results of the Stanford research on this grant were presented at the symposium.

Professor Breakwell attended the AAS Denver Conference, July 6-8, and co-authored two papers [Ref. 11,12].   He attended the Guidance Theory and Trajectory Analysis Meeting at ERC, Boston, September 28-29.

Professor Franklin has been elected to the Administrative Committee of the Group on Automatic Control of IEEE.   He is also chairman of the Theory Committee of the American Automatic Control Council and U.S. representative to the IFAC Theory Committee.

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